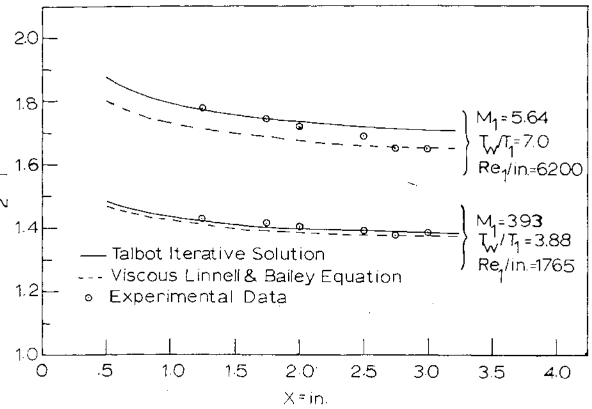

Fig. 1 Pressure ratio on a 5° half-angle cone.

Bailey[1] used the similarity concept to obtain the following expression for the pressure ratio at zero angle of attack,

$$P_c/P_1 = 1 + (\gamma M_1^2/2)(4 \sin^2\theta_c)(2.5 + 8\beta \sin\theta_c)/(1 + 16\beta \sin\theta_c) \quad (1)$$

Blick[2] has given expressions for temperature, velocity, and Mach number for cones using the parameter $M_1 \sin\theta_c$. They are,

$$T_c/T_1 = [1 + 0.35 (M_1 \sin\theta_c)^{1.5}] \text{ for } M_1 \sin\theta_c \leq 1.0 \quad (2a)$$

and, for $M_1 \sin\theta_c \geq 1.0$,

$$T_c/T_1 = [1 + \exp(-1 - 1.52 M_1 \sin\theta_c)][1 + (M_1 \sin\theta_c)^2/4] \quad (2b)$$

$$U_c/U_1 = \cos\theta_c [1 - \sin\theta_c/M_1]^{0.5} \quad (3)$$

$$M_c/M_1 = (U_c/U_1)(T_1/T_c)^{0.5} \quad (4)$$

To calculate the induced pressure on a cone due to the thickening of the boundary layer, one could use the method of Talbot[3] which involves an iteration on the flow variables at the edge of the boundary layer. Peter,[4] utilizing Eqs. (1-4) in a computer program based on Talbot's method, was able to obtain expressions for the boundary-layer displacement thickness gradient $d\delta^*/dx$ and the induced pressure on cones at zero angle of attack. His results are plotted in Fig. 1 vs the experimental data of King.[5]

By using Eqs. (1–4) and Talbot's[3] equation for boundary-layer displacement thickness,

$$(\delta^*/x) = (3.012/3^{1/2})(C/Re_2)^{1/3}\{(\pi/2)[T_W/T_2 - [\sigma(\gamma - 1)/4]M_2^2] - [1.0 + \sigma^{1/3} (T_W/T_2 - T_{AW}/T_2)]\} \quad (5)$$

it is possible to obtain a semiempirical equation for $d\delta^*/dx$ based on freestream properties and wall temperatures only. [Note that Eq. (5) is a function of properties at the edge of the boundary layer and hence are unknown at the start of an iterative solution.] This direct equation is

$$d\delta^*/dx = [0.342(T_W/T_1)^{1.3} + 0.08M_1^2][3.9M_1^{1/2}(Re_{1x})^{0.27} \times \{\theta_c^2 M_1 (Re_{1x})^{1/2} + 0.342(T_W/T_1)^{1.3} + 0.08M_1^2\}^{1/2}]^{-1} \quad (6)$$

Eq. (6) includes the transverse curvature effect of Hill[6] and is valid for a Prandtl number of 0.725.

The viscous solution for flow properties over slender cones can be obtained quickly by simply substituting θ_2 for θ_c in the inviscid equations [Eqs. (1–4)], where

$$\theta_2 = \theta_c + \tan^{-1}(d\delta^*/dx) \quad (7)$$

and where $d\delta^*/dx$ is obtained from Eq. (6). This means that the useful Linnell-Bailey equation [Eq. (1)] can be applied in either the viscous or inviscous regime.

Studies made over a Mach number range of 3–20, a Reynolds number range of 10^3–10^6, and wall temperature range of 1–9 showed that this simplified method gave pressure errors of less than 6% when compared with the lengthy iterative solutions.

A further comparison was made with the experimental data of King.[5] The results shown in Fig. 1 indicate that the simplified method (labeled the viscous Linnell and Bailey curve) falls within 3% of the experimental data.

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Reply to A. P. Cappelli

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Nomenclature

D	=	landing gear diameter
k	=	radius of gyration about center of gravity
k_e	=	effective spring constant per leg parallel to the vehicle's longitudinal axis
L_1	=	overturning radius ($0.5D \cos45°$ for 2-2 impact)
L_2	=	original height of center of gravity
LF	=	deceleration load factor based on earth gravity for landing on level surface with all legs crushing simultaneously
N	=	number of legs
V_v	=	vertical velocity
V_h	=	horizontal velocity
W_e	=	earth weight of vehicle
θ	=	lunar slope, negative for downhill landing
φ, $\dot{\varphi}$	=	vehicle attitude and attitude rate, positive nose up
μ	=	coefficient of friction

CAPPELLI[1] questions the discontinuity in the stability profile shown by the writer[2] since he has never observed any such discontinuity. There are basically two reasons for this. First, the stability profile is usually determined with a degree of accuracy (or rather lack of accuracy) as demonstrated by Cappelli in his Fig. 1.[1] There could very easily be a discontinuity in his profile of Fig. 1 at a vertical velocity of 8 fps, for example, without discovering it, since no analytical results were obtained at this velocity. In fact, even if re-

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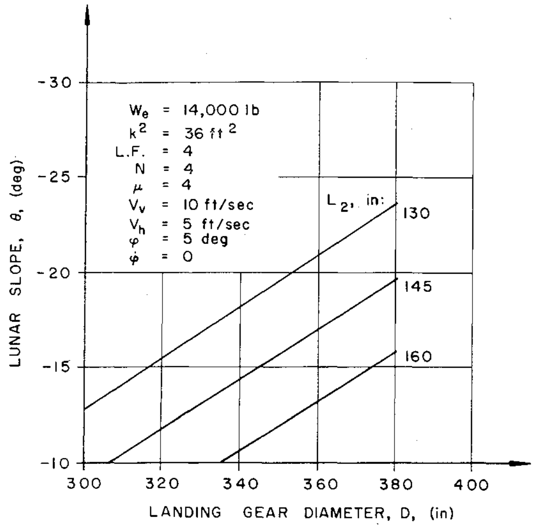

Fig. 1 Landing gear diameter vs lunar slope, downhill landing, 2-2 impact, method of Ref. 5.

sults were obtained at this velocity, the discontinuity could still be missed if it occurred at, say, 8.5 fps. Secondly, since the method described by Cappelli[3] fails to compute the proper forces on the vehicle during a purely rigid body rotation, he has no way of observing the effect of changing modes from a purely rigid body rotation to a purely free-flight condition after crushing stops. The discontinuity effect, when it does exist, appears to be of second order and does not seriously affect the stability profile. The discontinuity found by the writer[2] is the only such discontinuity so far observed and it was discovered quite by accident while varying vertical velocity at a constant horizontal velocity.

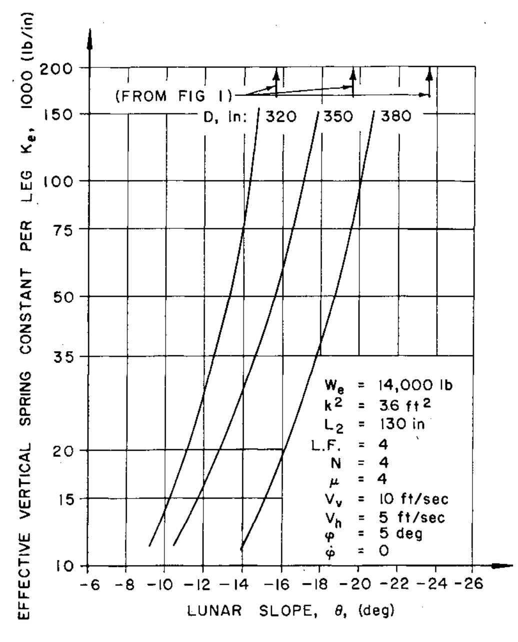

Fig. 2 Lunar slope vs effective vertical spring constant per leg, downhill landing, 2-2 impact; method of Ref. 7.

Since the writer has stated[2] that the method of Ref. 3 leads to optimistic results, Fig. 1 and Fig. 2 of Cappelli's reply[1] show comparisons of his theory with experimental drop tests. Very good agreement is shown between theory and experiment in his Fig. 1 with the theory being somewhat conservative rather than optimistic. The writer will not comment on his Fig. 2 concerning the Bendix Corporation drop tests except to admit to some surprise upon viewing a stability profile that intersects the origin. However, since the pitch attitude for these tests was a function of the initial velocity vector, the instability at the origin becomes understandable. In a private communication, Raymond J. Black, Bendix Corporation, has pointed out that the test results correspond to some very early Bendix work. The model was constructed mainly of wood with Teflon sleeve shock absorbers and was quite rigid. Their test fixture has now been changed so that the horizontal and vertical velocity, pitch angle, and angular velocity at the instant of impact can be chosen in any combination. The writer has the utmost respect for the work being done by the Bendix Corporation in the field of touchdown dynamics as demonstrated by a recent article.[4]

Returning to Cappelli's Fig. 1,[1] in view of the excellent agreement between theory and experiment, it would appear that Cappelli has demonstrated the validity of the analytical method. This is not the case, however, and leads to the most important point to be made in this comment. Cappelli's original article[3] concerned full-scale vehicles landing under the influence of lunar gravity. It was under these conditions that the writer found Cappelli's method leads to optimistic results.[2] Cappelli's reply[1] concerns scale model vehicles landing under the influence of earth gravity. There is a difference and it deserves some discussion.

The method of Ref. 5 was formulated in late 1962 as a means for investigating touchdown dynamics of logistic supply vehicles. The method includes the effects of crushing and sliding and was found to be quite conservative (requiring about 30% larger landing-gear diameters) compared to a simplified method based upon momentum-energy principles which was in use at the time. The method of Ref. 5, however, does not account for the effects of elasticity in the vehicle's structure nor for a variable landing-gear diameter during the touchdown motion in the case of vehicles with hinged legs. Subsequent to the development of the method of Ref. 5, the writer found the method to be very optimistic compared to results of touchdown dynamics analyses obtained by the Space Technology Laboratories[6] for full-scale vehicles landing on the lunar surface. Upon review of both methods, the writer believes the correct answer is somewhere in between. The method of Ref. 6 accounts for the elasticity effect, but does not account for any hinged leg effect. Thus, if the method of Ref. 5 gives optimistic results for full-scale vehicles landing on the moon and is also conservative compared to Cappelli's method, then certainly Cappelli's method is even more optimistic.

The writer has improved upon the method of Ref. 5 to include the effects of elasticity in the structure as well as hinged legs.[7] The method has been programed for digital computation by John D. Capps, Computation Laboratory, Marshall Space Flight Center. The extent of optimism in the method of Ref. 5 can be observed by comparing results from Fig. 1 with Fig. 2. Results from Fig. 1, obtained by the method of Ref. 5, corresponding to a c.g. height of 130 in., are shown by arrows in Fig. 2. These arrows indicate that the method of Ref. 5 essentially corresponds to infinite effective spring constants in the landing-gear legs. As seen from Fig. 2, there is serious loss in stability as the effective spring constant is reduced. For example, the method of Ref.

5 indicates that with a landing-gear diameter of 320 in. the vehicle is stable for slopes up to 15.6°. However, to achieve this same capability using a vertical spring constant per leg of 18,000 lb/in. requires a landing-gear diameter of 380 in., an increase of about 19%. Keeping the diameter of 320 in. and the effective vertical spring constant per leg of 18,000 lb/in. (an order-of-magnitude estimate for full scale vehicles), the vehicle is stable on slopes of only 11° or less. Thus, the elasticity effect is seen to be very important, an effect for which both the writer's method of Ref. 5 and Cappelli's method of Ref. 3 fails to account.

Since most scale model drop test vehicles seem to be rather rugged and stiff, the effective vertical spring constant per leg is generally large as compared to the value necessary for proper dynamic scaling. Assuming a $\frac{1}{6}$ scale model, the effective vertical spring constant should be only $\frac{1}{6}$ of the full scale value. Thus, although a particular method may agree favorably with experimental drop tests of rather stiff models, the method may actually be quite optimistic for full-scale vehicles landing on the moon.

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Erratum: "Experimental Convective Heat-Transfer Measurements"

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[AIAA J. 2, 2046–2047 (1964)]

CONVECTIVE heating comparisons between experiment and theory are given in Fig. 4 of the paper. Unfortunately typographical errors in Ref. 1 were propagated into these comparisons. The corrected comparisons are shown in Fig. 1.

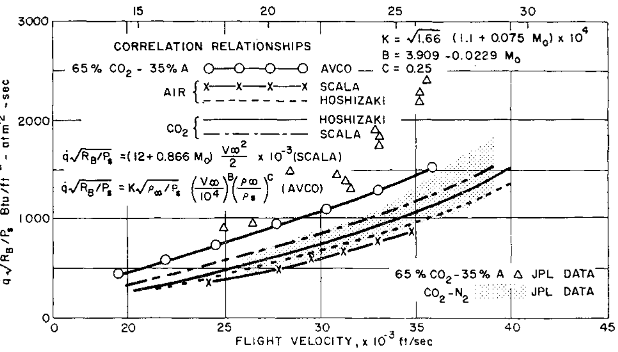

Fig. 1 Comparison of theoretical and experimental results.

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